Taylor Polynomial Practice - 11/9/16

1. Find the 4th order Taylor polynomial for $f(x) = e^{2x}$ at $x = 0$.

   Solution: We need to take 4 derivatives:

$$f'(x) = 2e^{2x}$$
$$f''(x) = 4e^{2x}$$
$$f'''(x) = 8e^{2x}$$
$$f^{(4)}(x) = 16e^{2x}$$

   Since $e^0 = 1$, we have

$$T_4(x) = 1 + \frac{2}{1!}x + \frac{4}{2!}x^2 + \frac{8}{3!}x^3 + \frac{16}{4!}x^4.$$

2. Find T_5 for $f(x) = \sin(x)$ at $x = \frac{\pi}{2}$.

   Solution: We need to take 5 derivatives:

$$f'(x) = \cos(x)$$
$$f''(x) = -\sin(x)$$
$$f'''(x) = -\cos(x)$$
$$f^{(4)}(x) = \sin(x)$$
$$f^{(5)}(x) = \cos(x)$$

   Since $\sin\left(\frac{\pi}{2}\right) = 1$ and $\cos\left(\frac{\pi}{2}\right) = 0$, then

$$T_5(x) = 1 - \frac{1}{2!}\left(x - \frac{\pi}{2}\right)^2 + \frac{1}{4!}\left(x - \frac{\pi}{2}\right)^4.$$

3. Find T_5 for $f(x) = \sin(x)$ at $x = 0$.

   Solution: We can use the same derivatives as we did for the previous problem. Since $\sin(0) = 0$ and $\cos(0) = 1$, then

$$T_5(x) = \frac{1}{1!}x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5.$$

4. Find the 4th order Taylor polynomial for $f(x) = \cos(2x)$ at $x = 0$.

Solution: We need to take 4 derivatives:

$$
\begin{aligned}
f'(x) &= -2\sin(2x) \\
f''(x) &= -4\cos(2x) \\
f'''(x) &= 8\sin(2x) \\
f^{(4)}(x) &= 16\cos(2x)
\end{aligned}
$$

Then

$$T_4(x) = 1 - \frac{4}{2!}x^2 + \frac{16}{4!}x^4.$$

5. Find the 2nd order Taylor polynomial for $f(x) = \ln(1 + x^2)$ at $x = 0$.

Solution: We need to take 2 derivatives:

$$
\begin{aligned}
f'(x) &= 2x\frac{1}{1+x^2} \\
f''(x) &= \frac{2}{1+x^2} + 2x(2x)\frac{1}{1+x^2}
\end{aligned}
$$

Then

$$T_2(x) = 0 + \frac{0}{1!}x + \frac{2}{2!}x^2 = x^2.$$